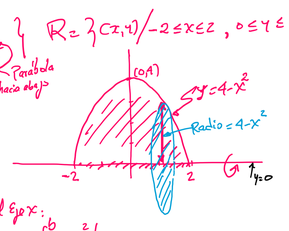
EJERCICIOS

viernes, 17 de julio de 2020 6:09 a. m.

Ej (1) $y=0$, $y=4-x^2$ (Parábola hacia abajo) $\}$ $R=\{(x,y)/-2\le x\le 2,\ 0\le y\le 4-x^2\}$

Cortes: $4-x^2=0$, $x^2=4$, $x=2$, $x=-2$ (Eje X)



(a) Rotación de R alrededor del Eje X:

$$V=\pi\int_{-2}^{2}(4-x^2)^2dx=2\pi\int_0^2(4-x^2)^2dx=2\pi\int_0^2(16-8x^2+x^4)dx$$

PAR

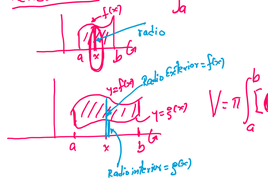
$$V=2\pi\left\{16x-\frac{8x^3}{3}+\frac{x^5}{5}\right\}_0^2=2\pi\left\{16(2)-\frac{8}{3}(2)^3+\frac{(2)^5}{5}\right\}$$

$$V=2\pi\left\{32-\frac{64}{3}+\frac{32}{5}\right\}=2\pi\left\{\frac{32\times15-64\times5+32\times3}{15}\right\}$$

$$V=2\pi\left\{\frac{256}{15}\right\}=\boxed{\frac{512\pi}{15}}$$ RTA. Unids³. de Volumen

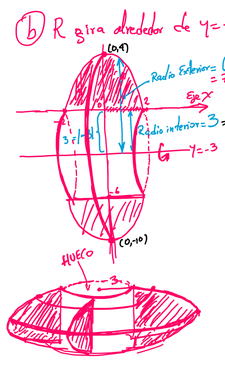
(a) R gira en torno al Eje X:

Método Disco: $V=\pi\int_a^b(f(x))^2dx$



$$V=\pi\int_a^b\left[(f(x))^2-(g(x))^2\right]dx$$

(b) R gira alrededor de $y=-3$



$V_T=2V_1$ (Por Simetría de la Figura)

$$V_1=\pi\int_0^2\left[(7-x^2)^2-3^2\right]dx=\pi\int_0^2\left[49-14x^2+x^4-9\right]dx=\pi\int_0^2\left[40-14x^2+x^4\right]dx$$

(Radio Ext)² (Radio Int)²

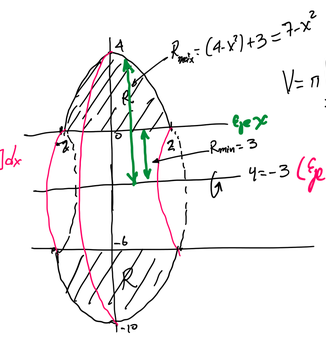
$$V_1=\pi\left[40x-\frac{14x^3}{3}+\frac{x^5}{5}\Big|_0^2\right]=\pi\left[80-\frac{14(8)}{3}+\frac{32}{5}\right]$$

$$=\pi\left[\frac{(80)(15)-14(40)+96}{15}\right]=\pi\left[\frac{1200-560+96}{15}\right]$$

$$\boxed{V_1=\pi\left[\frac{736}{15}\right]}$$

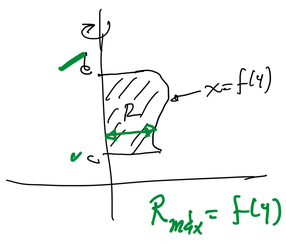
RTA

$$V_T=2V_1=\boxed{\frac{1472}{15}\pi}$$ Unids³ de Volumen



$$V=\pi\int_{-2}^{2}\left[\underbrace{(7-x^2)^2-3^2}_{\text{F. PAR}}\right]dx$$

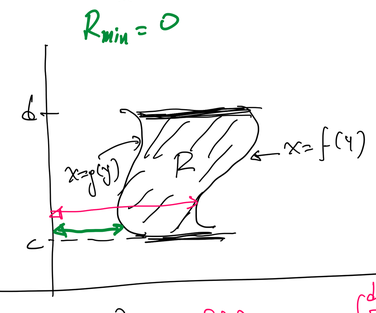
Nota:



$R_{max}=f(y)$

$R_{min}=0$

$$V=\pi\int_c^d(f(y))^2dy$$



$R_{máx}=f(y)$

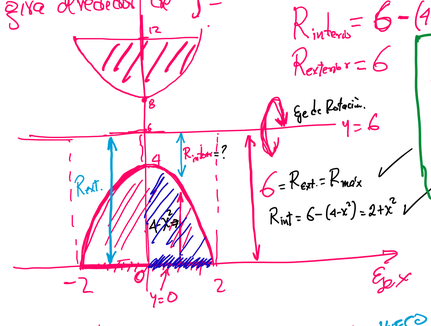
$R_{min}=g(y)$

$$V=\pi\int_c^d\left[(f(y))^2-(g(y))^2\right]dy$$

(c) R gira alrededor de $y=6$

$R_{interior}=6-(4-x^2)=2+x^2$

$R_{exterior}=6$



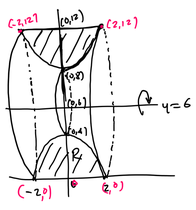
$6=R_{ext}=R_{máx}$

$R_{int}=6-(4-x^2)=2+x^2$

$$V=2\pi\int_0^2\left[6^2-(2+x^2)^2\right]dx=2\pi\int_0^2(36-4-4x^2-x^4)dx$$

$4+4x^2+x^4$

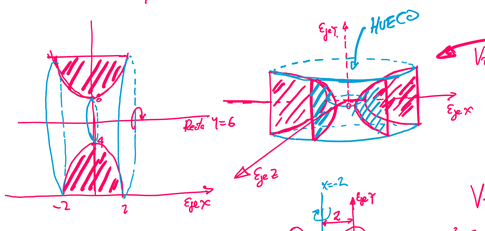
(R_exterior)² (R_interior)²



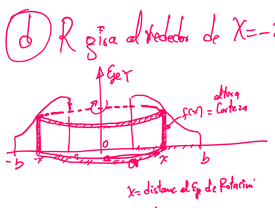
$$V=2\pi\left[32x-\frac{4x^3}{3}-\frac{x^5}{5}\Big|_0^2\right]=2\pi\left[64-\frac{32}{3}-\frac{32}{5}\right]$$

$$=2\pi\left[\frac{64(15)-32(5)-32(3)}{15}\right]=2\pi\left[\frac{960-160-96}{15}\right]$$

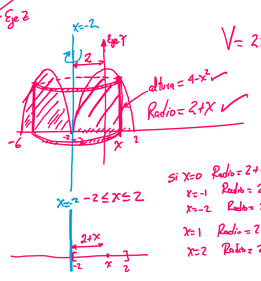
$$=2\pi\left(\frac{704}{15}\right)=\boxed{\frac{1408}{15}\pi}$$ Unid³. Volumen.



(d) R gira alrededor de $X=-2$



$$V=2\pi\int_a^b x\cdot f(x)\,dx$$



$$V=2\pi\int_{-2}^{2}(2+x)(4-x^2)dx=2\pi\int_{-2}^{2}(8-2x^2+4x-x^3)dx$$

$$=2\pi\left[8x-\frac{2x^3}{3}+2x^2-\frac{x^4}{4}\Big|_{-2}^{2}\right]$$

Si $x=0$ Radio $=2+0=2$

$x=1$ Radio $=2-1=1$

$x=-2$ Radio $=2-2=0$

$x=1$ Radio $=2+1=3$

$x=2$ Radio $=2+2=4$

$$=2\pi\left\{\left(16-\frac{16}{3}+\frac{8-4}{4}\right)-\left(-16+\frac{16}{3}+\frac{8-4}{4}\right)\right\}$$

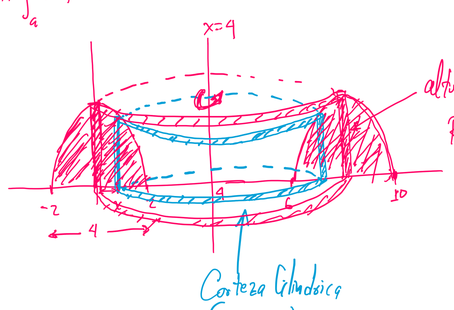
$$=2\pi\left\{\left(20-\frac{16}{3}\right)-\left(-12+\frac{16}{3}\right)\right\}$$

$$=2\pi\left\{32-\frac{32}{3}\right\}=2\pi\left\{\frac{96-32}{3}\right\}$$

$$=2\pi\left\{\frac{64}{3}\right\}=\boxed{\frac{128\pi}{3}}$$ RTA

Usando Méto. Corteza Cilíndrica

(e)



altura $=4-x^2$

Radio $=4-x$

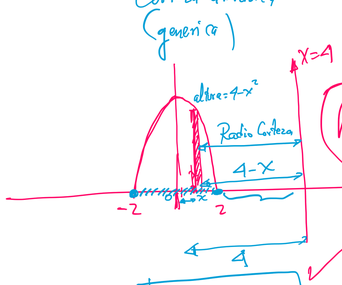
$$V=2\pi\int_{-2}^{2}(4-x)(4-x^2)d$$

$$=2\pi\int_{-2}^{2}[16-4x^2-4x-x^3]dx$$

$$=2\pi\left\{16x-\frac{4x^3}{3}-2x^2-\frac{x^4}{4}\Big|_{-2}^{2}\right\}$$

$=$ ✓ RTA

Corteza Cilíndrica (genérica)



Análisis Para la Fórmula del Radio

$\boxed{\text{Radio}=4-x}$ y $-2\le x\le 2$

es OK.

Si $x=-2$ Radio $=4-(-2)=6$

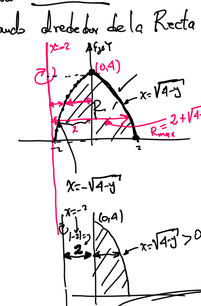
Si $x=0$ Radio $=4-0=4$

$x=2$ Radio $=4-2=2$

(d) Usando Método Disco.

R rotando alrededor de la Recta Vertical $x=-2$

$\int_c^d$ $c=0$, $d=4$

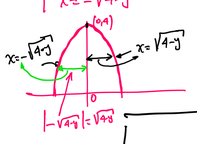


$y=4-x^2$

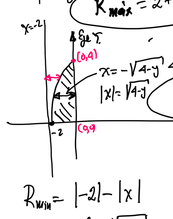
$x^2=4-y$

$x=\sqrt{4-y}$

$x=-\sqrt{4-y}$



$R_{máx}=2+\sqrt{4-y}$



$R_{min}=|-2|-|x|$

$=2-\sqrt{4-y}$

$$V_T=\pi\int_0^4\left[\left(2+\sqrt{4-y}\right)^2-\left(2-\sqrt{4-y}\right)^2\right]dy$$

$R_{máx}$ R_{min}

$$V=\pi\int_0^4\left[\left(4+4\sqrt{4-y}+4-y\right)-\left(4-4\sqrt{4-y}+4-y\right)\right]dy$$

$$=\pi\int_0^4\left[\cancel{4}+4\sqrt{4-y}+\cancel{4}-\cancel{y}-\cancel{4}+4\sqrt{4-y}-\cancel{4}+\cancel{y}\right]dy$$

$$=\boxed{8\pi\int_0^4(4-y)^{1/2}dy}=-8\pi\int_4^0u^{1/2}du=8\pi\int_0^4u^{1/2}du=8\pi\frac{u^{3/2}}{3/2}\Big|_0^4=\frac{16\pi}{3}(4)^{3/2}=\frac{16}{3}(8)\pi=\boxed{\frac{128}{3}\pi}$$ Unid³ de Volumen

$(4)^{3/2}=\sqrt{4^3}=\sqrt{64}=8$

RESP

Méto. Disco

$y=1/2$

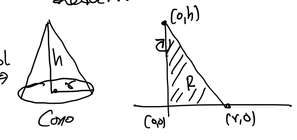
$\boxed{u=4-y}$ Cuando $y=0$ se tiene q $u=4$; " $y=4$ " " $u=0$

$du=-dy \rightarrow -du=dy$

Ejercicios (Quiz) (Proximo Viernes) Hallar las Fórmulas de Volumen o Deducir

✓ Volumen Esfera $=\frac{4}{3}\pi R^3$

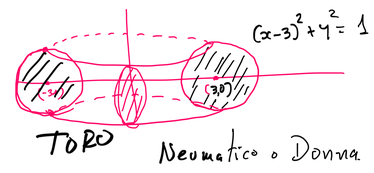
✓ Volumen Cono $=\boxed{\pi r^2h}$ Vol.



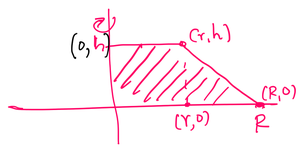
✓ Volumen Toro ✓

✓ Volumen Esfera Hueca

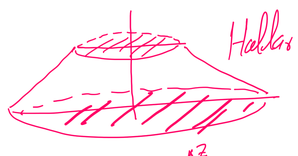
✓



TORO Neumatico o Donna

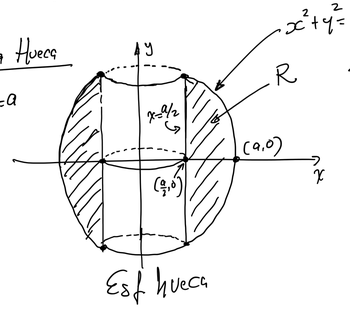


Hallar V.

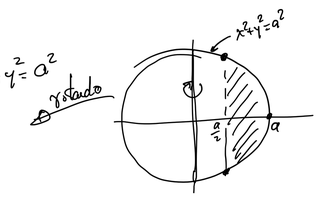


Esfera Hueca

Radio Esfera = a



Esf hueca

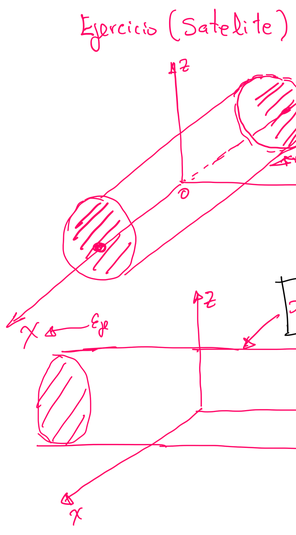


Ejercicio (Satelite) Ec. Cilindro

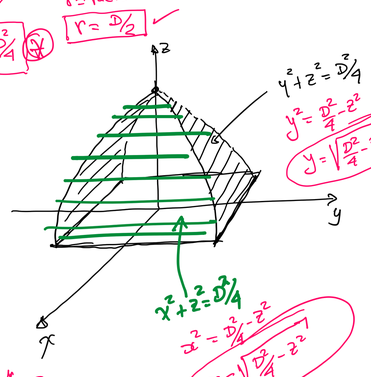
$x^2+y^2=r^2$ (z libre)

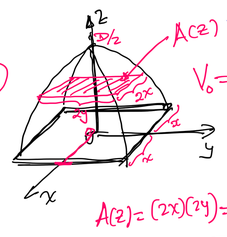
r = radio

$\boxed{r=D/2}$



$x^2+y^2=D^2/4$ (✓)





$A(z)=D^2-4z^2$

$$V_0=\int_0^{D/2}A(z)dz=\int_0^{D/2}(D^2-4z^2)dz=D^3/3$$

$V_T=2V_0=\frac{2}{3}D^3$ = Volu Satelite

$A(z)=(2x)(2y)=4xy$

$A(z)=4\sqrt{\frac{D^2}{4}-z^2}\sqrt{\frac{D^2}{4}-z^2}$

$A(z)=4\left(\frac{D^2}{4}-z^2\right)=D^2-4z^2$

$$V_0=\int_0^{D/2}(D^2-4z^2)dz=D^2z-\frac{4z^3}{3}\Big|_0^{D/2}$$

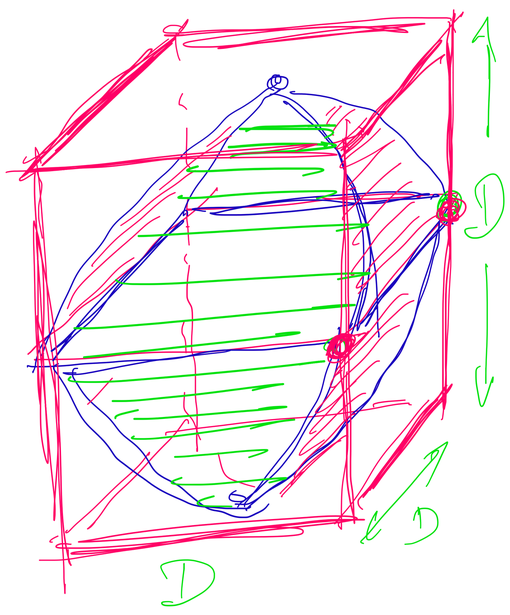
$$V_0=D^2\left(\frac{D}{2}\right)-\frac{4}{3}\left(\frac{D}{2}\right)^3$$

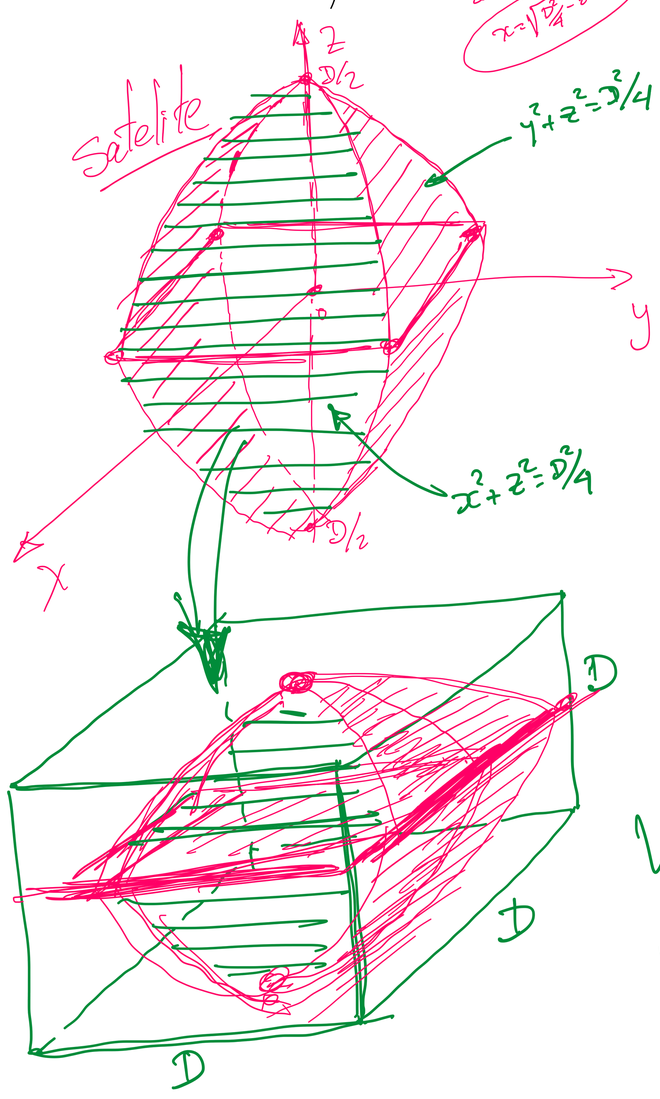
$$V_0=\frac{D^3}{2}-\frac{1}{6}D^3$$

$\left(\frac{D}{2}\right)^3=\frac{D^3}{8}$

$$V_0=\frac{3D^3}{6}-\frac{D^3}{6}=\frac{2}{6}D^3=\frac{D^3}{3}$$

$$\boxed{V_T=2V_0=\frac{2}{3}D^3}$$





V_0 = Volumen de la Parte Superior, es decir la Mitad del Volumen total del Satelite artificial

$V_{caja}=D^3$

$V_{caja}-V_T$ = Volumen q se Pierde

$D^3-\frac{2}{3}D^3=$ " "

$$\boxed{\frac{D^3}{3}=\text{Volumen q se Pierde}}$$