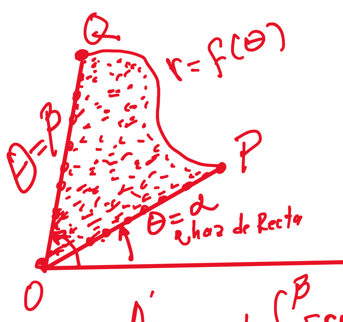
Ejemplos de areas entre curvas en C. Polares

jueves, 24 de marzo de 2022 6:34 a.m.



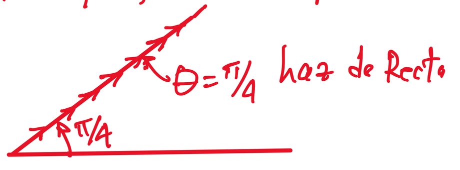
$\theta = \alpha$ = haz de Recta.

gráfica $= \{(r,\theta)/\theta = \alpha\}$

$= \{(r,\alpha)/r \geq 0\}$

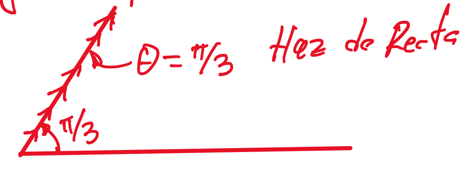
si $\alpha = \pi/4 \Rightarrow \theta = \pi/4$

gráfica $= \{(r,\pi/4)/r \geq 0\}$

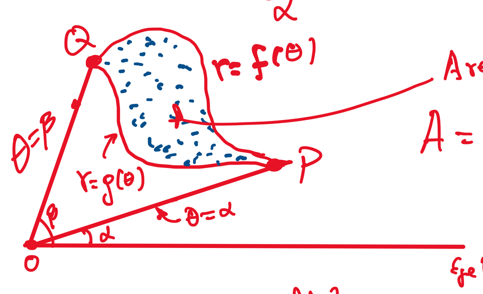


Si $\theta = \pi/3$

gráfica $= \{(r,\pi/3)/r \geq 0\}$



$$\text{Área} = \frac{1}{2}\int_\alpha^\beta [f(\theta)]^2 d\theta.$$

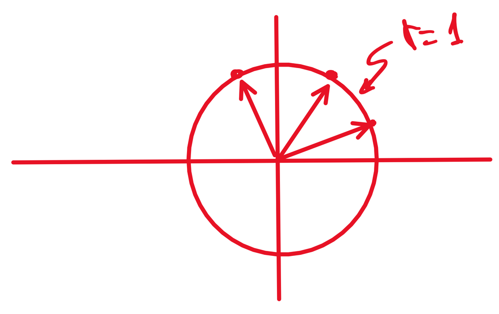


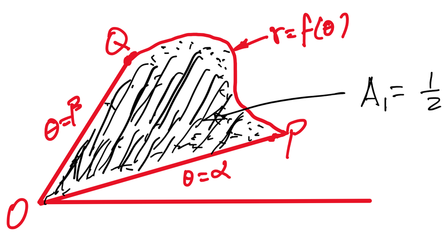
Area = ?

$$A = \frac{1}{2}\int_\alpha^\beta [(f(\theta))^2 - (g(\theta))^2] d\theta.$$

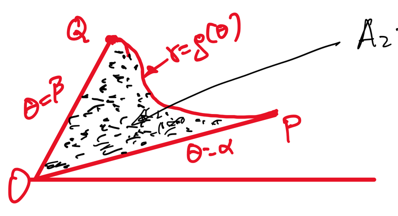
Si $r = 1$

graf $= \{(r,\theta)/r = 1\} = \{(1,\theta)/0 \leq \theta \leq 2\pi\}$





$$A_1 = \frac{1}{2}\int_\alpha^\beta [f(\theta)]^2 d\theta$$



$$A_2 = \frac{1}{2}\int_\alpha^\beta [g(\theta)]^2 d\theta$$

Area entre f y g $= A_1 - A_2$

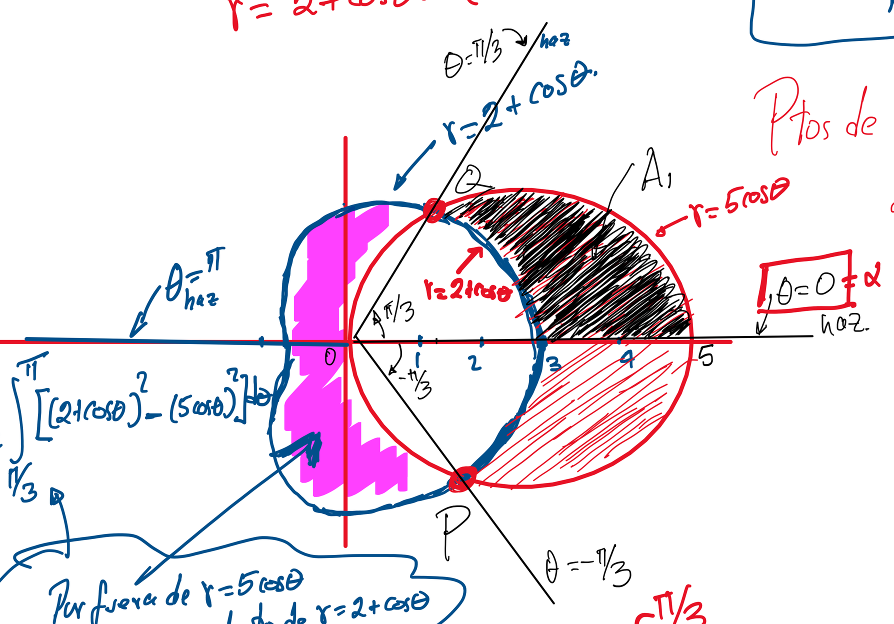
$$= \frac{1}{2}\int_\alpha^\beta (f(\theta))^2 d\theta - \frac{1}{2}\int_\alpha^\beta (g(\theta))^2 d\theta$$

$$\boxed{\text{Area entre } f \text{ y } g = \frac{1}{2}\int_\alpha^\beta [(f(\theta))^2 - (g(\theta))^2] d\theta}$$

Ejemplo: $r = 5\cos\theta$ (Círculo)

$r = 2 + \cos\theta$. (Caracol o Limaçon)

Hallar el área por fuera de $r = 2 + \cos\theta$ y por dentro de $r = 5\cos\theta$.



$$A_1 = \frac{1}{2}\int_{\pi/3}^{\pi} [(2+\cos\theta)^2 - (5\cos\theta)^2] d\theta$$

Por fuera de $r = 5\cos\theta$ y a su vez por dentro de $r = 2+\cos\theta$

Ptos de Intersección:

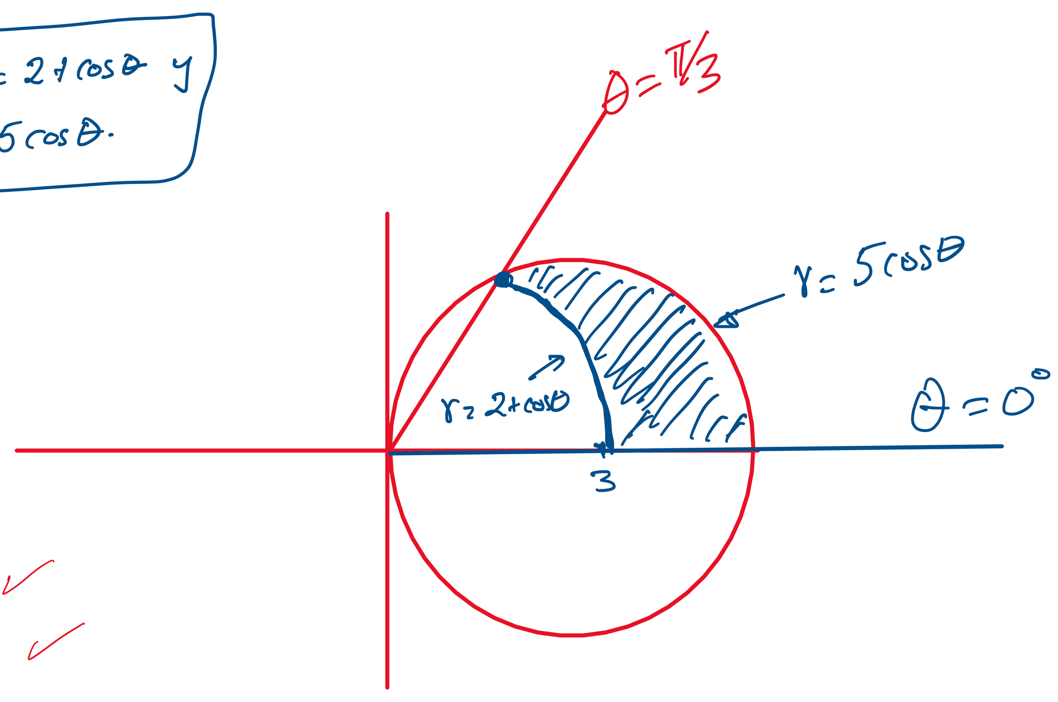
$2 + \cos\theta = 5\cos\theta$

$2 = 4\cos\theta$

$\cos\theta = \frac{1}{2}$

$\theta = \pi/3$ ✓

$\theta = -\pi/3$ ✓



$$A_1 = \frac{1}{2}\int_0^{\pi/3} [(5\cos\theta)^2 - (2+\cos\theta)^2] d\theta = \frac{1}{2}\int_0^{\pi/3} [25\cos^2\theta - (4 + 4\cos\theta + \cos^2\theta)] d\theta$$

$$= \frac{1}{2}\int_0^{\pi/3} [24\cos^2\theta - 4\cos\theta - 4] d\theta$$

Nota: $\int \cos^2\theta \, d\theta = \int \frac{1+\cos 2\theta}{2} d\theta = \frac{1}{2}\int d\theta + \frac{1}{2}\int \cos 2\theta \, d\theta$

$= \frac{\theta}{2} + \frac{1}{4}\operatorname{sen} 2\theta + C$

O si se quiere también es posible resolver

$$A_{Total} = \frac{1}{2}\int_{-\pi/3}^{\pi/3} [(5\cos\theta)^2 - (2+\cos\theta)^2] d\theta.$$

$$= \frac{1}{2}\left\{24\left(\frac{\theta}{2} + \frac{1}{4}\operatorname{sen} 2\theta\right) - 4\operatorname{sen}\theta - 4\theta \Big|_0^{\pi/3}\right\}$$

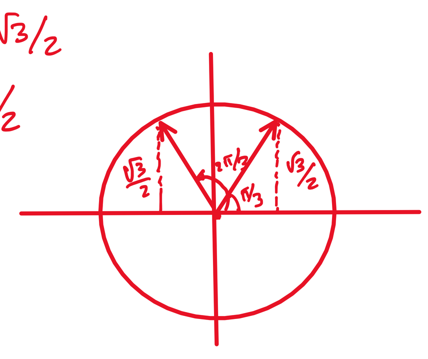
$$= \frac{1}{2}\left\{\cancel{12\theta} + 6\operatorname{sen} 2\theta - 4\operatorname{sen}\theta - \cancel{4\theta} \Big|_0^{\pi/3}\right\}$$

$$= \frac{1}{2}\left\{8\theta + 6\operatorname{sen} 2\theta - 4\operatorname{sen}\theta \Big|_0^{\pi/3}\right\}$$

$$= \frac{1}{2}\left\{8\pi/3 + 6\operatorname{sen}(2\pi/3) - 4\operatorname{sen}\pi/3\right\}$$

$\operatorname{sen}\pi/3 = \sqrt{3}/2$

$\operatorname{sen} 2\pi/3 = \sqrt{3}/2$



$$= \frac{1}{2}\left\{8\pi/3 + 6\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)\right\}$$

$$= \frac{1}{2}\left\{8\pi/3 + 2\left(\frac{\sqrt{3}}{2}\right)\right\} = \frac{1}{2}\left\{8\pi/3 + \sqrt{3}\right\}$$

$$A_1 = \frac{1}{2}\left(8\pi/3 + \sqrt{3}\right)$$

$$\boxed{A_{Total} = 8\pi/3 + \sqrt{3}}$$ RTA

Fórmulas Generales

1. $r = \pm K$ Son Círculo de Radio K
2. $\theta = \alpha$ (radianes) Haz de Rectas
3. $\left\{ \begin{array}{l} r = a\operatorname{sen} n\theta \\ r = a\cos n\theta \end{array} \right.$

Rosas de Pétalos

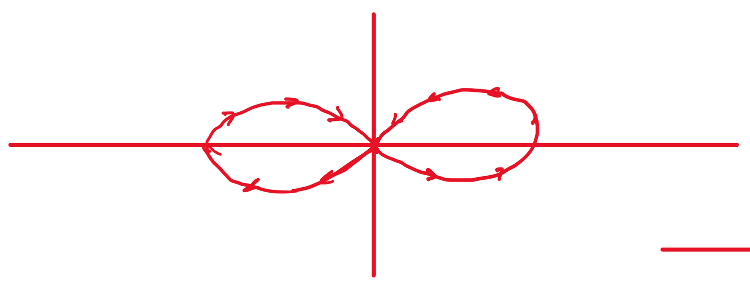
si n = impar, el # de Pétalos = n.

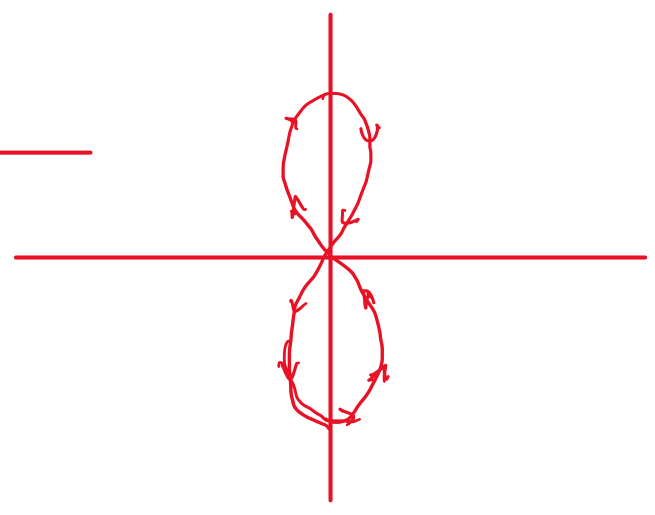
si n = Par, el # Pétalos = $2n$.

4. $r = a \pm b\cos\theta$
   $r = a \pm b\operatorname{sen}\theta$ } se llaman Caracoles o Limaçon.

Cuando $a = b$ la gráfica recibe el Nombre de Cardioide.

5. $r^2 = a^2\cos 2\theta$
   $r^2 = a^2\operatorname{sen} 2\theta$. } Se denominan lemniscatas





6. $r = K\theta$

Espiral de Arquímedes

$r = K$ Círculo

$r = K\theta$ Espiral