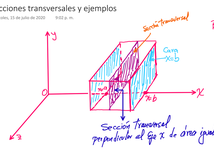
Secciones transversales y ejemplos

miércoles, 15 de julio de 2020 9:02 p. m.



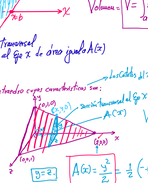
Sección Transversal perpendicular al Eje x de área igual a A(x)

Bloque
Cortes laterales $y=a$
$y=b$

$$\boxed{V_{\text{Volumen}} = V = \int_a^b A(x)\,dx}$$

Ej ① Hallar el Volumen de un tetraedro cuyas características son:

Las caras del triángulo de la sección son iguales



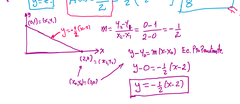
$\int u^n du = \frac{u^{n+1}}{n+1} + c$

$$V = \int_0^2 A(x)\,dx = \int_0^2 \frac{1}{8}(x-2)^2\,dx = \frac{1}{8}\cdot\frac{(x-2)^3}{3}\Big|_0^2 = \frac{1}{24}\left[(2-2)^3 - (0-2)^3\right] = \frac{1}{24}\left[8\right] = \boxed{\frac{1}{3}\text{ area}^3}$$

$\boxed{0=-2}$

$$\boxed{A(x) = \frac{y^2}{2} = \frac{1}{2}\left(-\frac{1}{4}(x-2)\right)^2 = \boxed{\frac{1}{8}(x-2)^2}}$$

= Area de la Sección Transversal en función de x $A(x)$



$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{0-1}{2-0} = -\frac{1}{2}$$

$y - 1 = -\frac{1}{2}(x-0)$

$(0,1,0)$, (x,y)

$y - y_0 = m(x - x_0)$ Ec. Pendiente

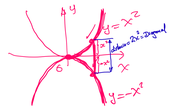
(x,y), $(x,0)$

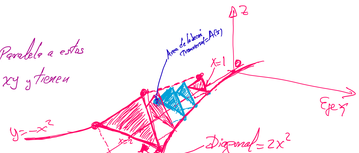
$y - 0 = -\frac{1}{2}(x-2)$

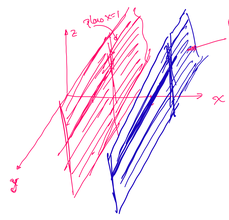
$$\boxed{y = -\frac{1}{2}(x-2)}$$

Ejercicio #32 (Página 509)

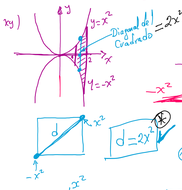
Un Sólido limitado por los planos $x=1$, $x=2$ es tal que toda sección recta Paralela a estos planos es un CUADRADO. Las diagonales de estos cuadrados están en el plano xy y tienen sus extremos en las Curvas $y = x^2$ (Parábola) $y = -x^2$ (Parábola). Hallar el Volumen del Sólido.



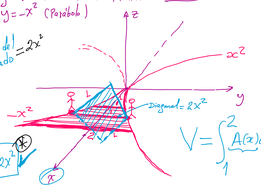




Piso (plano xy)



$$\boxed{d = 2x^2}$$



$$V = \int_1^2 A(x)\,dx = \int_1^2 2x^4\,dx = \frac{2x^5}{5}\Big|_1^2 = \frac{2}{5}\left[2^5 - 1^5\right] = \frac{2}{5}\left(\underbrace{32-1}_{31}\right)$$

$$\boxed{V = \frac{62}{5}\text{ unidades}^3}$$

$$A_{\text{Cuadrado}} = L^2 = \frac{d^2}{2} = \frac{(2x^2)^2}{2} = \frac{4x^4}{2} = \boxed{2x^4}$$

$A(x)$

Área de Cada Cuadrado en función de x

$d^2 = L^2 + L^2$ (Pitagoras)

$d^2 = 2L^2$

$$\boxed{L^2 = \frac{d^2}{2}}$$

Ejercicios:

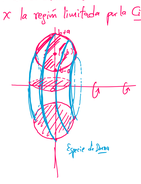
Página 513 (#22)

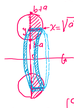
Por el método de la Corteza Cilíndrica, hallar el Volumen del toro generado al girar alrededor del eje x la región limitada por la Circunferencia $\boxed{x^2 + (y-b)^2 = a^2}$; $0 < a < b$

Centro de la Circunf. $= (0,b)$
radio $= a$

$x^2 = a^2 - (y-b)^2$

$x = \sqrt{a^2 - (y-b)^2}$





$x = \sqrt{a^2 - (y-b)^2}$ (altura)

y = distancia al Eje de Rotación.

$$V = 2\pi \int_c^d \underbrace{y}_{\text{distancia al eje}}\,\underbrace{g(y)}_{\text{altura}}\,dy$$

$$\boxed{V_{\text{Toro}} = 2V_1}$$

$$V_1 = 2\pi \int_{b-a}^{b+a} y\sqrt{a^2 - (y-b)^2}\,dy = 2\pi \int_{-\pi/2}^{\pi/2} (b + a\operatorname{sen}\theta)(a\cos\theta)(a\cos\theta\,d\theta) = 2\pi \int_{-\pi/2}^{\pi/2} (b + a\operatorname{sen}\theta)\,a^2\cos^2\theta\,d\theta = 2\pi\left\{\int_{-\pi/2}^{\pi/2} ba^2\cos^2\theta\,d\theta + \int_{-\pi/2}^{\pi/2} a^3\operatorname{sen}\theta\cos^2\theta\,d\theta\right\}$$

Sustitución Trigonométrica: $\boxed{y - b = a\operatorname{sen}\theta} \to \boxed{y = b + a\operatorname{sen}\theta} \to dy = a\cos\theta\,d\theta$

$\sqrt{a^2 - a^2\operatorname{sen}^2\theta} = a\sqrt{1 - \operatorname{sen}^2\theta} = a\cos\theta$

Límites de Integración: Cuando $y = b - a$ (reemplazo de $y = b - a$):

$\cancel{b} - a - \cancel{b} = a\operatorname{sen}\theta$

$-a = a\operatorname{sen}\theta$

$-1 = \operatorname{sen}\theta$

$\boxed{\theta = -\pi/2}$ límite inferior

Cuando $y = b + a$:

$y - b = a\operatorname{sen}\theta$

$\cancel{b} + a - \cancel{b} = a\operatorname{sen}\theta$

$a = a\operatorname{sen}\theta$

$1 = \operatorname{sen}\theta$

$\boxed{\theta = \pi/2}$ límite superior

$$V_1 = 2\pi\left\{ba^2\int_{-\pi/2}^{\pi/2}\left(\frac{1+\cos 2\theta}{2}\right)d\theta - a^3\int_{-\pi/2}^{\pi/2}\cos^2\theta\,(-\operatorname{sen}\theta\,d\theta)\right\}$$

$\int u^n\,du = \frac{u^{n+1}}{n+1} + C$

$u = \cos\theta \quad du = -\operatorname{sen}\theta\,d\theta$

$$= 2\pi\left\{ba^2\left[\frac{\theta}{2} + \frac{1}{4}\operatorname{sen}2\theta\right]_{-\pi/2}^{\pi/2} - a^3\frac{\cos^3\theta}{3}\Big|_{-\pi/2}^{\pi/2}\right\}$$

$$= 2\pi\left\{ba^2\left[\frac{\pi}{4} - \frac{(-\pi/2)}{2}\right] - \frac{a^3}{3}\left(\cos^3\pi/2 - \cos^3(-\pi/2)\right)\right\}$$

$$= 2\pi\left\{ba^2\left(\frac{\pi}{2}\right)\right\}$$

$$\boxed{V_1 = \pi^2 ba^2}$$

R+A.

$$V_{\text{Toro}} = 2V_1 = \boxed{2\pi^2 ba^2}$$

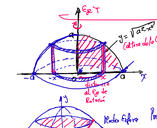
$\boxed{\text{Centro } (0,b) \quad \text{Radio } a}$

$(x-b)^2 + y^2 = a^2$

$C = (0,5)$
$b = 5$
radio $= a = 2$

$$x^2 + (y-5)^2 = 4$$

$$V = 2\pi^2(5)(2)^2 = \boxed{40\pi^2}$$

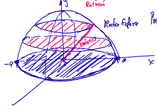


$y = \sqrt{a^2 - x^2}$

$y^2 = a^2 - x^2$

$x^2 + y^2 = a^2$ (Circunf. de radio a)

$$\boxed{V = 2\pi\int_0^a x\sqrt{a^2 - x^2}\,dx} = 2\pi\int_0^a (a^2 - x^2)^{1/2}\cdot x\,dx$$



Medio Esfera

Pregunta $V_{\text{esfera}} = ?$

$$= -\pi\int_0^a (a^2 - x^2)^{1/2}\cdot[-2x]\,dx$$

$\int u^n\,du = \frac{u^{n+1}}{n+1} + C$

$$= -\pi\left[2\frac{(a^2 - x^2)^{3/2}}{3}\Big|_0^a\right]$$

$$= -\pi\left[\frac{2}{3}(a^2 - a^2)^{3/2} - \frac{2}{3}(a^2 - 0)^{3/2}\right]$$

$$= \frac{2\pi}{3}(a^2)^{3/2} = \frac{2\pi}{3}a^3$$

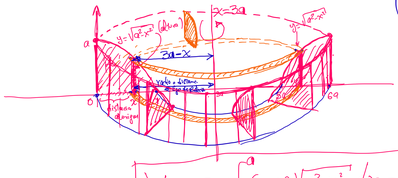
R+A. donde a = radio de la Esfera

V_{esfera}

$$V_{\text{Esfera}} = \frac{4\pi}{3}a^3$$

$$\boxed{V_{\text{esf}} = \frac{4}{3}\pi R^3}$$

fórmula del Volumen de una Esfera de radio R



$$\boxed{V = 2\pi\int_0^a \underbrace{(3a - x)}_{\text{distancia al Eje de Rotac.}}\sqrt{a^2 - x^2}\,dx = \text{Realizar el Cálculo}}$$