


3. Un péndulo consiste en una masa m conectada a una cuerda de longitud L . La masa se desplaza de modo que la cuerda forma un ángulo de θ_0 con la vertical, y luego se libera la masa describe un semiarco. Encuentre la expresión para la velocidad y la tensión de la cuerda cuando la masa pasa por la parte inferior.

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$


$$\vec{T} = T \hat{j} \quad \theta = 90^\circ$$

$$\vec{T} \cdot \vec{v} = 0$$

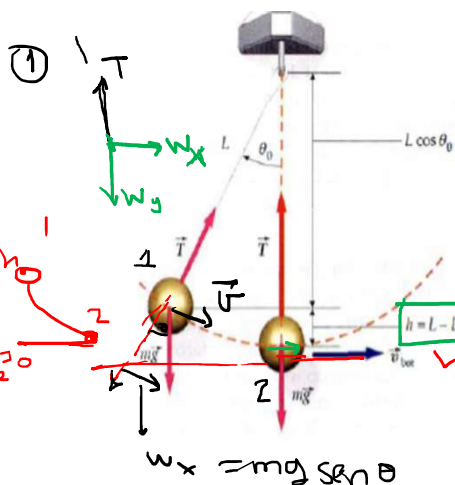
$$W_{nc} = 0$$

$$h_1 = h$$

$$v_1 = 0$$

$$v_2 = ?$$

$$h_2 = 0$$



$$W_{Ext} = 0$$

$$W_{nc} = \int \vec{T} \cdot d\vec{l}$$

$$d\vec{l} = \vec{v} dt$$

$$W_{nc} = \int_1^2 \vec{T} \cdot \vec{v} dt$$

$$F_{conservative} = W_x$$

$$\Delta E_{mec} = 0$$

$$E_{mec,1} = E_{mec,2}$$

$$U_{g1} + K_1 = U_{g2} + K_2$$

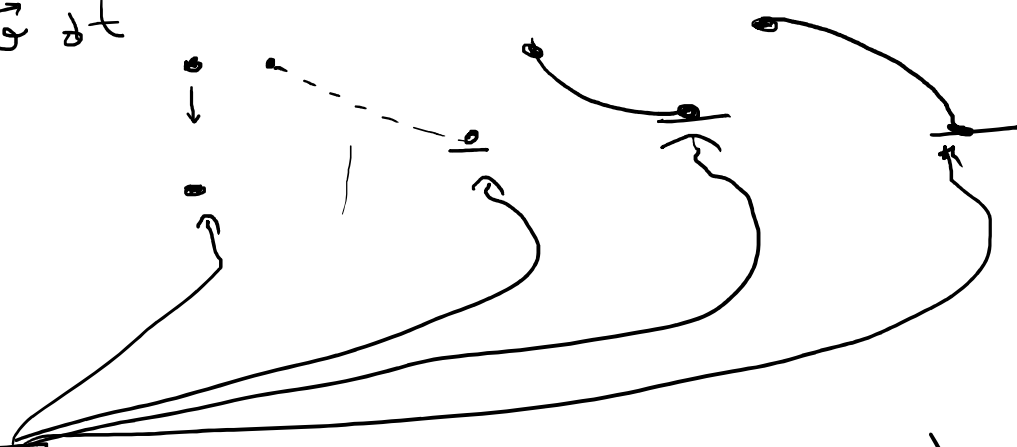
$$mgh_1 + \frac{1}{2}mv_1^2 = mgh_2 + \frac{1}{2}mv_2^2$$

$$\cos \theta = \frac{L}{L+h}$$

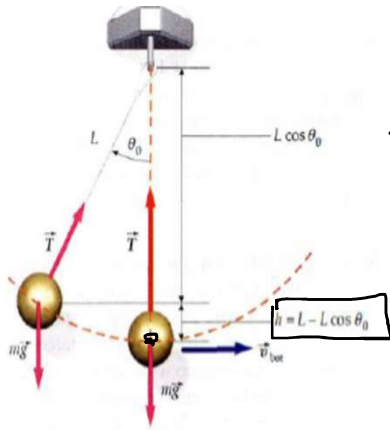
$$(L+h) \cos \theta = L$$

$$gh_1 = \frac{1}{2}v_2^2 \Rightarrow \sqrt{2gh_1} = v_2$$

F_c única que hace trabajo



$v_1 = 0$ (1)



$v_2 =$

$$h = L(1 - \cos \theta)$$

$$v_2 = \sqrt{2gh}$$

$$v_2 = \sqrt{2g(L - L \cos \theta)}$$

→ Cinemática
 $\sum F \Rightarrow \text{Constante} \Rightarrow \text{MUA}$

$$v_2^2 - v_1^2 = 2 \Delta y a_y$$

$$\frac{v_2^2}{2 \Delta y} = a_y = \frac{2gL(1 - \cos \theta)}{2L}$$

$$a_y = g(1 - \cos \theta)$$

$$T = ?$$

$$\sum F_y = ma_y$$

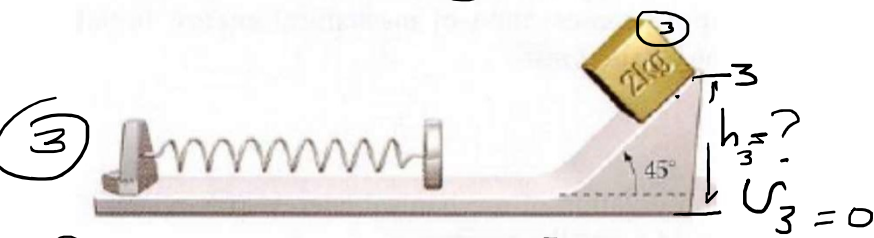
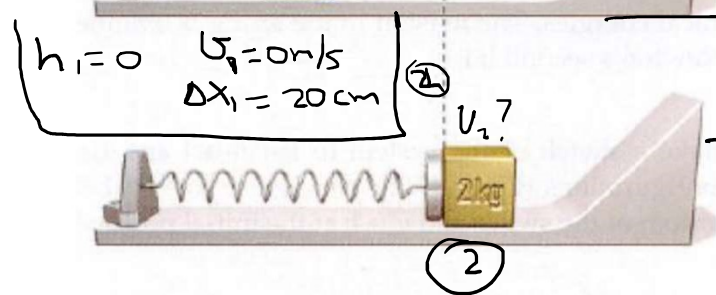
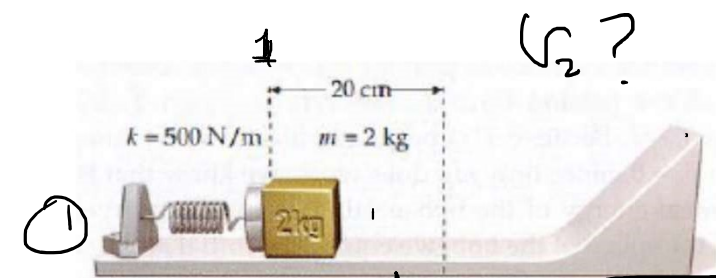
$$T - mg = ma_y$$

$$T = m(a_y + g)$$

$$T = m(g(1 - \cos \theta) + g)$$

$$T = mg(1 - \cos \theta + 1)$$

$$T = mg(2 - \cos \theta)$$



③ → ③ $h?$ $\Delta E_{mec} = 0$
 $E_{mec,2} = E_{mec,3}$

$$K_2 + U_{g,2} = K_3 + U_{g,3}$$

$$\frac{1}{2} m v_2^2 + m g h_2 = \frac{1}{2} m v_3^2 + m g h_3$$

$$\frac{1}{2} m v_2^2 = m g h_3$$

$$\Delta E_{mec} = 0$$

$$E_{mec,1} = E_{mec,2}$$

$$E_{mec,2} = E_{mec,3}$$

$$E_{mec,1} = E_{mec,3}$$

$$U_{r,1} + U_{g,1} + K_1 = U_{r,3} + K_3 + U_{g,3}$$

$$\frac{1}{2} k \Delta x_1^2 + m g h_1 + \frac{1}{2} m v_1^2 = \frac{1}{2} k \Delta x_3^2 + m g h_3 + \frac{1}{2} m v_3^2$$

$$\frac{1}{2} k \Delta x_1^2 = \frac{1}{2} m v_2^2$$

$$v_2 = \sqrt{\frac{k \Delta x_1^2}{m}} = \sqrt{\frac{500 \text{ N/m} \cdot (0.2 \text{ m})^2}{2 \text{ kg}}}$$

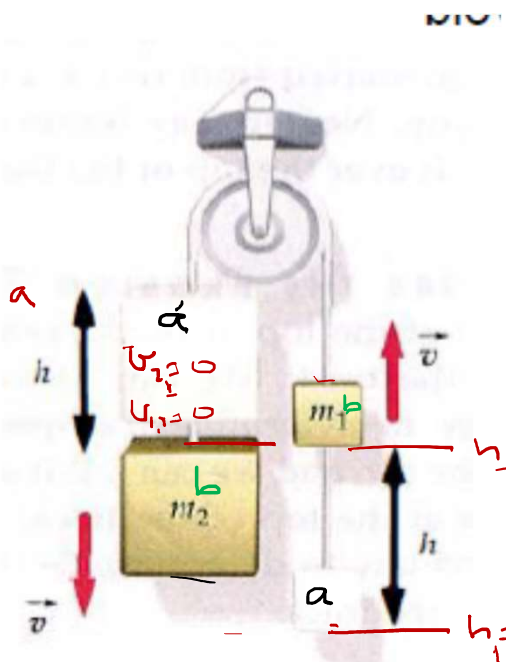
$$K_2 = \frac{1}{2} k \Delta x_1^2 = \frac{1}{2} \cdot 500 \text{ N/m} \cdot (0.2 \text{ m})^2$$

$$k = m g h$$

$$h = \frac{k}{m g}$$

$$v_2 = \sqrt{2 g h_3}$$





Hay solamente una fuerza conservativa F_G

$$\Delta E_{mec} = 0$$

$$E_{mec1} = E_{mec2}$$

$$K_1 + K_{1a} + K_{2a} + U_{g1a} + U_{g2a} = K_{1b} + K_{2b} + U_{g1b} + U_{g2b}$$

$$\frac{1}{2} m_1 \cancel{v_{a1}^2} + \frac{1}{2} m_2 \cancel{v_{a2}^2} + m_2 g \cancel{h_{a2}} + m_1 g \cancel{h_{a1}} =$$

$$\frac{1}{2} m_2 v_{2b}^2 + \frac{1}{2} m_1 v_{1b}^2 + m_2 g \cancel{h_{b2}} + m_1 g h_{1b}$$

$$h_{2a} = h$$

$$v_{2b} = v_{1b}$$

$$h_{1b} = h$$

$$m_2 g h_{2a} = \frac{1}{2} m_2 v_{2b}^2 + \frac{1}{2} m_1 v_{1b}^2 = m_1 g h_{1b}$$

$$m_2 g h = \frac{1}{2} m_2 v_{1b}^2 + \frac{1}{2} m_1 v_{1b}^2 = m_1 g h$$

$$m_2 g h - m_1 g h = \frac{1}{2} v_{1b}^2 (m_1 + m_2)$$

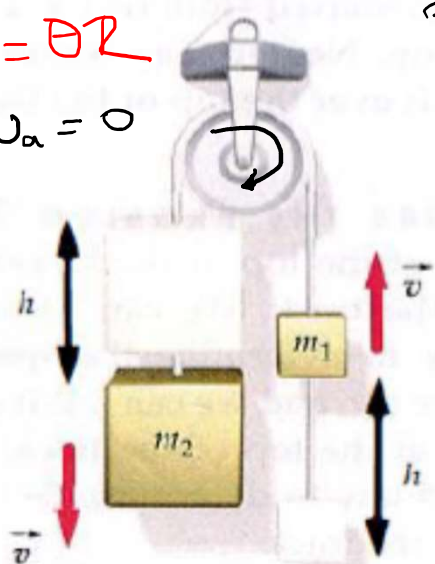
$$\sqrt{\frac{2 g h (m_2 - m_1)}{m_1 + m_2}} = v_{1b} = v_{2b}$$



$$\theta$$

$$S = \theta R$$

$$\omega_a = 0$$



Polea es un disco de radio R y masa M

$$\vec{\omega} = -\omega \hat{u}_\theta$$

$$I_D = \frac{1}{2} MR^2$$

$$K_{R_{D_a}} + K_1 + K_2 + U_{g_1} + U_{g_2} = K_{R_{D_b}} + K_1 + K_2 + U_{g_1} + U_{g_2}$$

$$\frac{1}{2} m_1 \cancel{v_a^2} + \frac{1}{2} m_2 \cancel{v_a^2} + \frac{1}{2} I \cancel{\omega^2} + m_1 g h_a + m_2 g h_a =$$

$$\frac{1}{2} m_1 v_b^2 + \frac{1}{2} m_2 v_b^2 + \frac{1}{2} I \omega_b^2 + m_1 g h_b + m_2 g h_b$$

K_T = Energía cinética

Traslación

$$\frac{1}{2} m v^2$$

$$\frac{1}{2} I \omega^2$$

$$K_R$$

K_R = Energía Rotacional.

Velocidad angular ω

masa \Rightarrow Momento de

I

$$\frac{\text{Inercia}}{(\omega = \frac{v}{R})}$$

$$\theta = R \omega$$

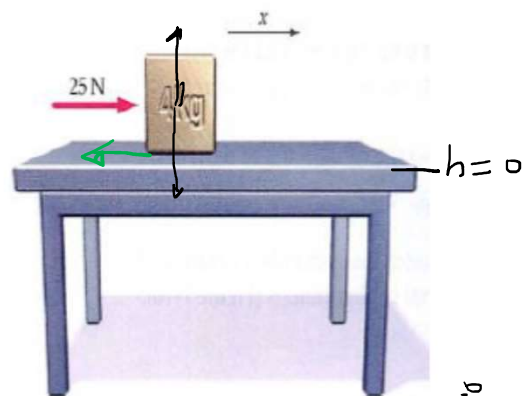
$$\frac{1}{2} v_b^2 (m_1 + m_2) + \frac{1}{2} I \omega_b^2 + m_1 g h = m_2 g h$$

$$\frac{1}{2} v_b^2 (m_1 + m_2) + \frac{1}{2} I \frac{v_b^2}{R^2} = m_2 g h - m_1 g h$$

$$\frac{1}{2} v_b^2 (m_1 + m_2) + \frac{1}{2} M R^2 \frac{v_b^2}{R^2} = g h (m_2 - m_1)$$

$$\frac{1}{2} v_b^2 (m_1 + m_2 + \frac{M}{2}) = g h (m_2 - m_1)$$

$$v_b = \sqrt{\frac{4 g h (m_2 - m_1)}{(2m_1 + 2m_2 + M)}}$$



6. Usted empuja una caja de 4 kilogramos, que está al principio descansando sobre una mesa horizontal, con una fuerza horizontal de 25 N se desplaza una distancia de 3 m. El coeficiente de fricción cinética entre la caja y la mesa es 0.35. Encuentre al trabajo externo sobre el sistema de mesa de bloque, (b) la energía disipada por la fricción, (c) la energía cinética final de la caja, y d) la velocidad de la caja, Si Usted empuja una caja desde el reposo.

$$E_{mec} = K + U_g + U_k$$

$$\Delta E_{mec} = \Delta K$$

$$f_f = \mu N$$

$$f_f = \mu_k mg (-\hat{i})$$

$$\Delta x = d \hat{i}$$

$$W_{ff} = -d \mu_k mg$$

$$W_{F_{ext}} = \vec{F}_{ext} \cdot \vec{\Delta x} = 25 N \times 3 m (\hat{i} \cdot \hat{i})$$

$$W_{ff} = -3 m \times 0.35 \times 4 kg \times 9.81 m/s^2 =$$

$$F_{nc} = \begin{matrix} 1. F_f \\ 2. F_{ext} \end{matrix} \quad W_{nc} = W_{ff} + W_{F_{ext}}$$

$$W_{nc} = \Delta E_{mec}$$

$$\Rightarrow W_{nc} = K_f - K_i$$

$$W_{F_{ext}} = 25 N \times 3 m = 75 Nm$$

$$W_{nc} = W_{ff} + W_{F_{ext}} =$$

$$W_{nc} = -41.20 J + 75 J = K_f \Rightarrow \frac{1}{2} m v_f^2 = W_{nc}$$

$$v_f^2 = \frac{2 W_{nc}}{m}$$

$$v_f = \sqrt{\frac{2 \times 33.8 J}{4 kg}}$$

$$v_f = 4.11 m/s$$

$$\vec{F}_{ext} = 25 N \hat{i}$$

$$m = 4 kg$$

$$d = \Delta x = 3 m \hat{i}$$

$$\mu_k = 0.35$$

$$W_{F_{ext}} ?$$

$$\text{Energía disipada por la fricción} = W_{ff}$$

$$-41.20 J = \text{Energía disipada por fricción}$$